Oxford Cambridge and RSA

Wednesday 5 June 2019 – Morning

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

You must have:
• Printed Answer Booklet

You may use:
• a scientific or graphical calculator

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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Solve the inequality $10x^2+x-2>0$. **[4]**

Factorising $10x^2+x-2 = (5x-2)(2x+1)$

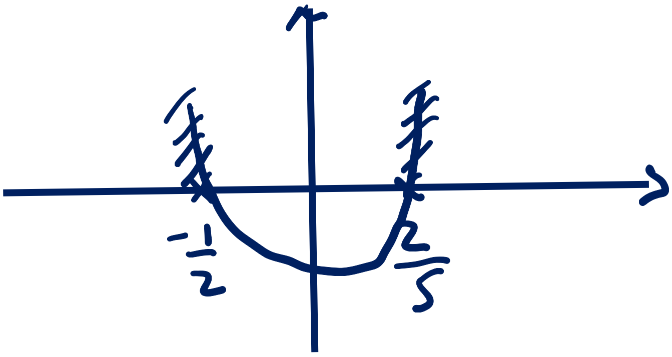
$(5x-2)(2x+1)>0$

Solve
sketch
Range

The roots of the equation are;

$(5x-2)(2x+1)=0$

$x=\frac{2}{5}$ $\quad x=-\frac{1}{2}$



→ From the diagram above the values of x that satisfy the inequality are;

$x<-\frac{1}{2}$ and $x>\frac{2}{5}$

2 The point A is such that the magnitude of $\overrightarrow{OA}$ is 8 and the direction of $\overrightarrow{OA}$ is 240°.

(a) **(i)** Show the point A on the axes provided in the Printed Answer Booklet. **[1]**

(ii) Find the position vector of point A.
Give your answer in terms of **i** and **j**. **[3]**

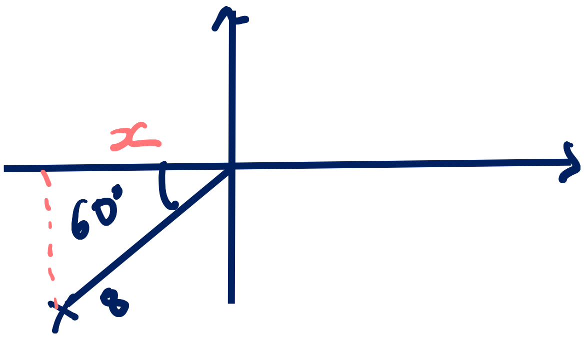
The point B has position vector 6**i**.

(b) Find the exact area of triangle AOB. **[2]**

The point C is such that $OABC$ is a parallelogram.

(c) Find the position vector of C.
Give your answer in terms of **i** and **j**. **[2]**

ai)



ii) Finding x and y components

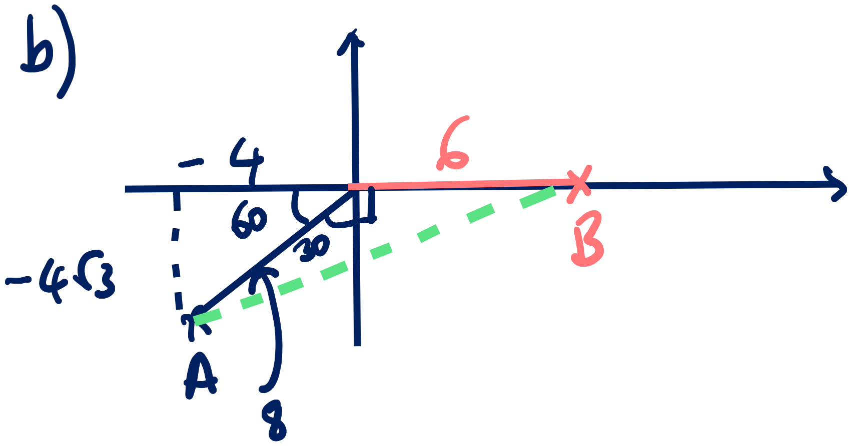
$x = 8\cos 60 = 4$

$y = 8\sin 60 = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$.

* Since they are in the 3rd quadrant both i and j components are negative *

$\Rightarrow -4i - 4\sqrt{3}j$

b)



* Area $= \frac{1}{2}ab\sin\theta$ *

$\Rightarrow \frac{1}{2} \times 8 \times 6 \times \sin(90+30)$

$= 12\sqrt{3}$ units^2

(c) $\Rightarrow 6i - (-4i - 4\sqrt{3}j)$ (as $\overrightarrow{AO} = \overrightarrow{CB}$)

$= 10i + 4\sqrt{3}j$.

3 The function f is defined by $f(x) = (x-3)^2 - 17$ for $x \geqslant k$, where k is a constant.

(a) Given that $f^{-1}(x)$ exists, state the least possible value of k. **[1]**

(b) Evaluate ff(5). **[2]**

(c) Solve the equation $f(x) = x$. **[3]**

(d) Explain why your solution to part **(c)** is also the solution to the equation $f(x) = f^{-1}(x)$. **[1]**

a) $x-3=0 \quad x=3 \quad \therefore \boxed{k=3}$

b) $f(5) = (5-3)^2-17 \Rightarrow (2)^2-17 \rightarrow 4-17$
$= -13$

$\Rightarrow$ -13 is not in the domain
so $f(-13)$ ie $ff(5)$ can not be defined.

(c) $(x-3)^2-17=x$.

$x^2-6x+9-17=x$

$x^2-6x-8=x$

$\underbrace{x^2-7x-8}=0$

factorising this;

$(x-8)(x+1)=0 \quad \therefore x=8, -1$

this is not a valid soln. as $x \geqslant 3$

$\therefore \boxed{x=8 \text{ only}}$.

d) $f(x)$ and $f^{-1}(x)$ are reflections on the line $y=x$, so the point of intersection must be on the line $y=x$.

4 Sam starts a job with an annual salary of £16 000. It is promised that the salary will go up by the same amount every year. In the second year Sam is paid £17 200.

(a) Find Sam's salary in the tenth year. **[2]**

(b) Find the number of complete years needed for Sam's **total** salary to first exceed £500 000. **[4]**

(c) Comment on how realistic this model may be in the long term. **[1]**

a) This is an arithmetic progression with;

$a = 16,000$

$d = 1200 \quad (17,200 - 16,000)$

$u_n = a + d(n-1) \quad \therefore \quad u_{10} = 16,000 + 1200(10-1)$

$= £26,000$

b) $S_n > 500,000$

$S_n = \frac{n}{2}[2a + d(n-1)]$

$\frac{n}{2}[2(16,000) + 1200(n-1)] > 500,000$

$\frac{n}{2}[32,000 + 1200n - 1200] > 500,000$

$n[30,800 + 1200n] > 1,000,000$

$\underbrace{1200n^2 + 30,800n - 1,000,000}_{} > 0$

equating this to zero gives; $n = 18.8, (\text{or } -44.4)$

$\therefore n = 19$

c) Unrealistic - as Sam is unlikely to stay in the same role that long.

5 A curve has equation $x^3 - 3x^2y + y^2 + 1 = 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{6xy - 3x^2}{2y - 3x^2}$. **[4]**

(b) Find the equation of the normal to the curve at the point (1, 2). **[4]**

a) Using implicit differentiation;

$$\frac{dy}{dx} = 3x^2 - 3x^2 \cdot \frac{dy}{dx} - 6xy + 2y \cdot \frac{dy}{dx} = 0.$$

Bringing like terms together:

$$2y \cdot \frac{dy}{dx} - 3x^2 \cdot \frac{dy}{dx} = 6xy - 3x^2$$

$$\frac{dy}{dx}(2y - 3x^2) = 6xy - 3x^2$$

$$\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2} \quad \text{as required.}$$

b) $$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=2}} = \frac{6(1)(2) - 3(1)^2}{2(2) - 3(1)^2} = \frac{9}{1} = 9.$$ ← gradient of tangent

$\therefore$ gradient of normal $= -\frac{1}{9}$.

$$y - y_0 = m(x - x_0)$$

$$y - 2 = -\frac{1}{9}(x - 1)$$

$$y = -\frac{1}{9}x + \frac{1}{9} + 2 \Rightarrow y = -\frac{1}{9}x + \frac{19}{9}$$

or

$$x + 9y = 19$$

6 Let $f(x) = 2x^3 + 3x$. Use differentiation from first principles to show that $f'(x) = 6x^2 + 3$. **[6]**

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$f(x+h) - f(x) =$

$[2(x+h)^3 + 3(x+h)] - [2x^3 + 3x]$

From above

$[2(x^3 + 3x^2h + 3xh^2 + h^3) + 3x + 3h] - 2x^3 - 3x$

Expanding

$\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 + \cancel{3x} + 3h - \cancel{2x^3} - \cancel{3x}$.

$\Rightarrow 6x^2h + 6xh^2 + 2h^3 + 3h$

$$\frac{f(x+h) - f(x)}{h} = \frac{6x^2h + 6xh^2 + 2h^3 + 3h}{h}$$

$$\Rightarrow \frac{\cancel{h}(6x^2 + 6xh + 2h + 3)}{\cancel{h}} = 6x^2 + 6xh + 2h + 3$$

BUT $f'(x)$ is when $h \to 0$

$\therefore f'(x) = 6x^2 + 6x(0) + 2(0) + 3$

$= 6x^2 + 3$ as required.

7 **In this question you must show detailed reasoning.**

A sequence $u_1, u_2, u_3 \ldots$ is defined by $u_n = 25 \times 0.6^n$.

Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^{N} u_n < 10^{-4}$. **[8]**

This is a geometric progression with;

$a = 15$
$r = 0.6$

Since summations start at n=1, so the progression formula is adjusted

Sum to infinity

$\frac{a}{1-r} = \frac{15}{1-0.6} = \frac{75}{2}$

Sum from $1 \to N$

$\frac{a(1-r^n)}{1-r} = \frac{15(1-0.6^N)}{1-0.6}$

$= \frac{75}{2}(1-0.6^N)$

$\frac{75}{2} - \frac{75}{2}(1-0.6^N) < 10^{-4}$

$\frac{75}{2} \cdot 0.6^N < 10^{-4}$

$\Rightarrow 0.6^N < 10^{-4} \times \frac{2}{75} \qquad \Rightarrow 0.6^N < \frac{1}{375000}$

$N > \frac{\log(1/375{,}000)}{\log(0.6)} \qquad \Rightarrow N > 25.125\ldots$

hence $N = 26$

8 A cylindrical tank is initially full of water. There is a small hole at the base of the tank out of which the water leaks.

The height of water in the tank is x m at time t seconds. The rate of change of the height of water may be modelled by the assumption that it is proportional to the square root of the height of water.

When $t = 100$, $x = 0.64$ and, at this instant, the height is decreasing at a rate of $0.0032\,\text{ms}^{-1}$.

(a) Show that $\frac{dx}{dt} = -0.004\sqrt{x}$. **[2]**

(b) Find an expression for x in terms of t. **[4]**

(c) Hence determine at what time, according to this model, the tank will be empty. **[2]**

a) $\frac{dx}{dt} \propto \sqrt{x}$

$\therefore \frac{dx}{dt} = k\sqrt{x}$

0.64

-0.0032

Because height is decreasing

$-0.0032 = k\sqrt{0.64}$

$k = \frac{-0.032}{\sqrt{0.64}}$

$= -0.004$

$\therefore \frac{dx}{dt} = -0.004\sqrt{x}$ as required.

b) $\int \frac{dx}{\sqrt{x}} = \int -0.004\, dt$.

$\int x^{-1/2}\, dx = \int -0.004\, dt$

$\frac{x^{1/2}}{1/2} = -0.004t + c$

$2x^{1/2} = -0.004t + c$

$x^{1/2} = -0.002t + c$

when $x = 0.64$ $t = 100$

$(0.64)^{1/2} = -0.002(100) + c$

$c = 1$

$x^{1/2} = -0.002t + 1$

$x = (1 - 0.002t)^2$

(c) $x = 0$

$1 = 0.002t$

$t = 500s$

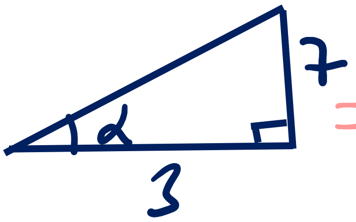
9 **(a)** Express $3\cos 3x + 7\sin 3x$ in the form $R\cos(3x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

(b) Give full details of a sequence of three transformations needed to transform the curve $y = \cos x$ to the curve $y = 3\cos 3x + 7\sin 3x$. **[4]**

(c) Determine the **greatest** value of $3\cos 3x + 7\sin 3x$ as x varies and give the smallest positive value of x for which it occurs. **[2]**

(d) Determine the **least** value of $3\cos 3x + 7\sin 3x$ as x varies and give the smallest positive value of x for which it occurs. **[2]**

9)



$R\cos(3x-\alpha)$
$= R\cos 3x\cos\alpha + R\sin 3x \sin\alpha$

$R^2 = 7^2 + 3^2$

$R^2 = 58$

$R = \sqrt{58}$

Rsinα

$\tan\alpha = \frac{7}{3}$. $\alpha = 1.17$

Rcosα

$\therefore \sqrt{58}\cos(3x - 1.17)$

b) → Stretch in the y direction by s.f $\sqrt{58}$

→ Translation in the x-direction by 1.17

→ Stretch in the x direction by s.f 1/3.

c) The greatest value cos can take is 1

$\therefore$ the greatest value $= \sqrt{58}$.

This occurs when

$3x - 1.17 = 0$

$x = 0.39$

d) The least value cos can take is -1

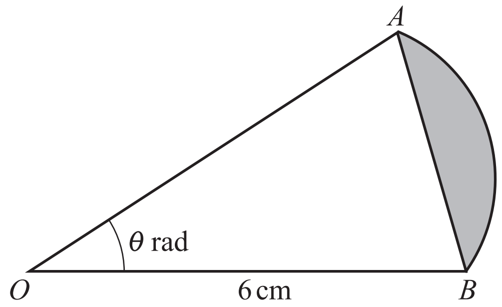
$\therefore$ the least value $= -\sqrt{58}$

This occurs when

$3x - 1.17 = \pi$

$x = \frac{\pi + 1.17}{3} = 1.44$

10



The diagram shows a sector AOB of a circle with centre O and radius 6 cm.
The angle AOB is θ radians.
The area of the segment bounded by the chord AB and the arc AB is $7.2\,\text{cm}^2$.

(a) Show that $\theta = 0.4 + \sin\theta$. **[3]**

(b) Let $\mathrm{F}(\theta) = 0.4 + \sin\theta$.

By considering the value of $\mathrm{F}'(\theta)$ where $\theta = 1.2$, explain why using an iterative method based on the equation in part **(a)** will converge to the root, assuming that 1.2 is sufficiently close to the root. **[2]**

(c) Use the iterative formula $\theta_{n+1} = 0.4 + \sin\theta_n$ with a starting value of 1.2 to find the value of θ correct to 4 significant figures.
You should show the result of each iteration. **[3]**

(d) Use a change of sign method to show that the value of θ found in part **(c)** is correct to 4 significant figures. **[3]**

a) Area of segment = Area of sector − Area of triangle.

$\Rightarrow$ Area of sector

$\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times \theta = 18\theta$

Area of triangle

$\frac{1}{2}ab\sin\theta = \frac{1}{2}\times 6\times 6\times \sin\theta = 18\sin\theta$.

$18\theta - 18\sin\theta = 7.2 \Rightarrow \frac{18(\theta - \sin\theta)}{18} = \frac{7.2}{18}$

$\theta - \sin\theta = 0.4 \Rightarrow \theta = 0.4 + \sin\theta$ as required.

b) $F'(1.2) = \cos(1.2)$

$|F'(1.2)| < 1 \quad \therefore$ iteration will converge.

c) $\theta_{n+1} = 0.4 + \sin\theta_n$

$\underline{n=1}$

$\theta_2 = 0.4 + \sin\theta_1 = 0.4 + \sin(1.2) = 1.3320\ldots$

$\underline{n=2}$

$\theta_3 = 0.4 + \sin\theta_2 = 0.4 + \sin(1.3320\ldots) = 1.3716\ldots$

Repeating this process gives

$\Rightarrow \quad 1.3802, \quad 1.3819, \quad 1.3822, \quad 1.3823$

$\Rightarrow \quad \theta = 1.382 \quad (4sf)$

d) Upper bound $= 1.3825$

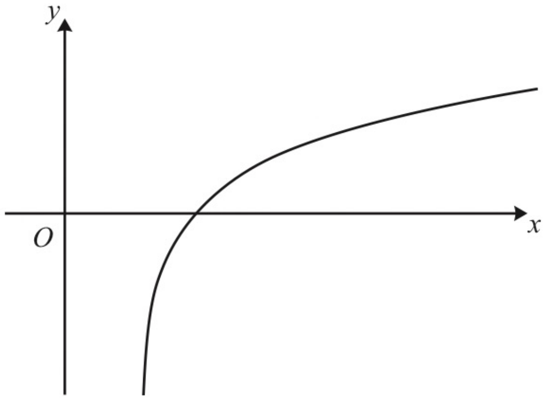
Lower bound $= 1.3815$

$f(1.3825) = -0.000637$

$f(1.3815) = 0.000175$

$\rightarrow$ The change in sign in the interval $1.3815 < \theta < 1.3825$, it shows us $\theta = 1.382$ to 4.s.f.

11



The diagram shows part of the curve $y = \ln(x-4)$.

(a) Use integration by parts to show that $\int \ln(x-4)\,dx = (x-4)\ln|x-4| - x + c$. **[5]**

(b) State the equation of the vertical asymptote to the curve $y = \ln(x-4)$. **[1]**

(c) Find the total area enclosed by the curve $y = \ln(x-4)$, the x-axis and the lines $x = 4.5$ and $x = 7$. Give your answer in the form $a\ln 3 + b\ln 2 + c$ where a, b and c are constants to be found. **[4]**

a) $\int 1 \cdot \ln(x-4)\,dx$.

$u = \ln(x-4)$ $\quad v' = 1$

$u' = \dfrac{1}{x-4}$ $\quad v = x$

Integration by parts formula.

$uv - \int v u'\,dx$.

$$x \ln|x-4| - \int \frac{x}{x-4}\,dx.$$

need to turn this into a proper fraction

$$\frac{x}{x-4} \equiv A + \frac{B}{x-4}$$

$$\frac{A(x-4)+B}{x-4} \equiv \frac{x}{x-4}$$

$$A(x-4)+B = x$$

let $x=4$

$4 = B.$

let $x=0$

$-4A+B=0$

But $B=4$

$-4A+4=0$

$A=1$

$\therefore \Rightarrow 1 + \dfrac{4}{x-4}$.

$$x\ln|x-4| - \int 1 + \frac{4}{x-4}\, dx$$

$$\Rightarrow \underbrace{x\ln|x-4| - x + 4\ln|x-4|}_{\text{Bringing like terms together.}} + C$$

$(x-4)\ln|x-4| - x + C$ as required.

b) $x = 4$

c) $\left|\int_5^7 \ln(x-4)\, dx\right| - \int_{4.5}^{5} \ln(x-4)\, dx$ (as below x axis)

$$\Big[(x-4)\ln|x-4| - x\Big]_5^7 - \Big[(x-4)\ln|x-4| - x\Big]_{4.5}^{5}$$

$$(3\ln 3-7)-(1\underbrace{\ln 1}_{=0}-5)-(1\underbrace{\ln 1}_{=0}-5)+\left(-\tfrac{1}{2}\ln\tfrac{1}{2}-\tfrac{9}{2}\right)$$

$$=3\ln 3-\tfrac{1}{2}\ln\tfrac{1}{2}-\tfrac{3}{2}$$

12 A curve has equation $y = a^{3x^2}$, where a is a constant greater than 1.

(a) Show that $\frac{dy}{dx} = 6xa^{3x^2} \ln a$. **[3]**

(b) The tangent at the point $(1, a^3)$ passes through the point $(\frac{1}{2}, 0)$.

Find the value of a, giving your answer in an exact form. **[4]**

(c) By considering $\frac{d^2y}{dx^2}$ show that the curve is convex for all values of x. **[5]**

a) let $3x^2 = u \Rightarrow \frac{du}{dx} = 6x$.

$y = a^u \Rightarrow \frac{dy}{du} = a^u \ln a$.

<u>using chain rule</u>

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= a^u \ln a \times 6x$$

$3x^2$

$$= a^{3x^2} \ln a \times 6x$$

$$= 6x\,a^{3x^2} \ln a \quad \text{as required.}$$

b) $\left.\frac{dy}{dx}\right|_{x=1} = 6(1)\, a^{3(1)^2} \ln a$

$= 6a^3 \ln a.$

$y - y_0 = m(x - x_0)$

$y - a^3 = 6a^3 \ln a\,(x-1)$

$y = 6a^3 \ln a\, x - 6a^3 \ln a + a^3.$

passes through $(\frac{1}{2}, 0)$

$0 = 6a^3 \ln a (\frac{1}{2}) - 6a^3 \ln a + a^3$

$0 = 3a^3 \ln a - 6a^3 \ln a + a^3.$

$0 = a^3 - 3a^3 \ln a.$

$0 = a^3(1 - 3\ln a)$

$a^3 = 0 \quad a = 0$

OR.

$1 - 3\ln a = 0 \Rightarrow \ln a = \frac{1}{3} \quad a = e^{1/3}$

c) $\frac{dy}{dx} = 6x\,a^{3x^2} \ln a$ but $a = e^{1/3}$

$= 6x\,e^{x^2} \ln e^{1/3} = 6x\,e^{x^2} \times \frac{1}{3}$

$= 2x\,e^{x^2}$

$\frac{d^2y}{dx^2} = ?$

using product rule ;

$2x \cdot 2x\,e^{x^2} + 2e^{x^2}$

$= 4x\,e^{x^2} + 2e^{x^2}$

$\rightarrow e^{x^2} > 0$ for all values of x, so

$2e^{x^2} > 0$.

$\rightarrow x^2 \geq 0$ for all values of x, so

$4x^2 e^{x^2} \geq 0$

$\therefore \quad 4x^2 e^{x^2} + 2e^{x^2} \geq 0$

$\therefore \frac{d^2y}{dx^2} > 0$ for all x. $\therefore$ the curve is always convex